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Michal Franta and Jan Vlček*

Abstract

Inflation at Risk provides a coherent description of the risks associated with an inflation outlook. This paper explores the practical applicability of this approach in central banks. The method is applied to Czech inflation to highlight issues related to short data sample. A set of quantile regressions with a non-crossing quantiles constraint is estimated using monthly data from the year 2000 onwards, and the model's in-sample fit and out-of-sample forecasting performance are then assessed. Furthermore, we discuss the Inflation at Risk estimates in the context of several historical events and demonstrate how the approach can inform monetary policy. The estimation results suggest the presence of nonlinearities in the Czech inflation process, which are related to supply-side pressures. In addition, it appears that regime changes have occurred recently.

JEL Codes: E31, E37, E52.

Keywords: Inflation dynamics, inflation risk, quantile regressions.

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1. Introduction

Recently many central banks have recognized that focusing solely on mean or mode inflation forecasts may not be sufficient. The reasons for a more nuanced view include asymmetric shock distributions, possibly with fat tails; nonlinearities in macroeconomic relationships; structural breaks; and regime changes.[1] Moreover, the risk of 'tail' events such as deflation and excessive inflation is of interest to policymakers in itself.

This paper offers such a nuanced perspective on the inflation outlook. Drawing on the recently developed concept of 'Inflation at Risk', it demonstrates how this can be applied to interpret inflation dynamics, assess inflation risks and their balance, and inform monetary policymakers. Thus, the paper contributes to the ongoing general discussion in central banks about how to deal with uncertainty related to inflation.

To illustrate its application in a real-life context, the approach is applied to the Czech Republic using monthly data from January 2000 to December 2024. The tails of the inflation outlook distribution, conditional on the current macroeconomic variables, are analyzed using a set of quantile regressions with a non-crossing quantiles constraint. Based on these estimates, the links between macroeconomic variables and the inflation forecast distribution are examined. Furthermore, the historical evolution of Czech inflation is discussed in terms of the inflation outlook distribution dynamics and balance of inflation risks.

The Czech Republic was chosen to demonstrate the broad applicability of the 'Inflation at Risk' approach. This country is representative of a short data sample, thus broadening the set of central banks that could find the approach useful. The issue of a short data sample is thoroughly discussed in the paper to highlight the questions it raises.

The estimation results suggest that the traditional Phillips curve relationship is important for both the center and the tails of inflation outlook distribution. However, the relationship exhibits non-linear features. It is found that the asymmetry of the inflation outlook distribution relates mainly to the growth of producer prices, which serve as a proxy for supply shocks. A high inflation environment alters the way in which supply shocks are transmitted to other prices.

The estimated tails of the Czech inflation outlook distribution indicate periods when the substantial mass of the distribution shifted below or above the inflation target, or when the tails exhibited long-term trends. For example, the upper tail steadily decreased during the zero lower bound period and when unconventional monetary policy measures were launched in the Czech economy, implying an increased probability of very low inflation. Next, ex-post observed inflation residing in the upper tail of an inflation outlook distribution suggests that a regime change in inflation dynamics has recently occurred.

It turns out that quantile regressions provide a superior in-sample fit of the inflation outlook distribution tails in comparison to tails based on conditional mean forecasts and the variance of OLS residuals (i.e. fan charts). Thus, the assessment of inflation risks and inflation dynamics improves when current macroeconomic variables are considered and third and higher moments of the inflation outlook distribution are accounted for. On the other hand, accounting for third or higher moments

[1] See, for example, Mishkin (2008) for such considerations in the context of financial disruptions.

is not crucial for forecasting. Models that explicitly model the variance of the inflation outlook are often sufficient in terms of their out-of-sample predictive ability.

The paper draws on several strands of literature. The first strand involves the empirical literature examining inflation uncertainty and risks.[2] Inflation at Risk is a model-based approach, as opposed to news- and survey-based indicators of inflation uncertainty and risk (see Cascaldi-Garcia et al., 2023, for a comprehensive survey).

In general, the Inflation at Risk literature finds that the conditional distribution of future inflation changes its shape over time. Furthermore, the drivers of inflation risk vary by country and evolve over time. López-Salido and Loria (2024) show that financial conditions vary with the lower tail of the conditional distribution of future inflation in both the United States and the Euro Area. Tighter financial conditions are associate with a higher probability of very low inflation. Moreover, the upper tail remains unaffected, implying that downside inflation risks stemming from the financial sector rise when financial conditions tighten. Similarly, López-Salido and Loria (2024) found that oil prices affect the distribution of inflation outlook asymmetrically. In addition to financial conditions impacting downside inflation risks, the post-Covid period emphasized the importance of fiscal policy and supply-side disruptions relating to the upper tail of an inflation outlook distribution. Makabe and Norimasa (2022) demonstrate that unit labor costs and real government spending pose significant upward inflation risks in three advanced economies: the United States, Canada and Germany. Furthermore, unlike import prices, these upward risks are not short-lived.

Banerjee et al. (2024) distinguishes between advanced and emerging market economies and found that nominal exchange rate depreciation leads to larger increases in the upper quantiles than in the lower quantiles of the four-quarters-ahead inflation distribution. This holds true for emerging market economies, but not for advanced economies. Additionally, tight financial conditions mitigate both upside and downside inflation risks, leaving the distribution mode unchanged in both emerging and advanced economies.

The dominant methodology employed in the ‘Inflation at Risk’ literature is based on quantile regressions with the determinants motivated by the Phillips curve.[3] Often, the entire distribution is fitted using a skewed t-distribution. Time-varying parameter quantile regressions are also used to examine the changing role of inflation risk factors over time (Korobilis et al., 2021).

Our study also contributes to the literature focusing on how policymakers (should) react to the abovementioned findings. This literature challenges the traditional view that policymakers can disregard the occurrence of economic shocks. This concept is known as ‘certainty equivalence’ (Theil, 1958).

The majority of papers adopt a positive stance, empirically assessing the risk management approach to the Federal Reserve's monetary policy under Greenspan. Killian and Manganelli (2008) formalize

[2] The distinction between risk and uncertainty lies in the ability to assign a probability to the possible outcomes of a variable.

[3] An alternative to the quantile regression approach is to simulate the entire predictive distribution using macroeconomic models. For example, Carriero et al. (2023) simulate the predictive distributions of output growth, inflation and unemployment using a BVAR model, demonstrating that this approach provides a description of tail risks with an accuracy comparable to that of the quantile regression approach. Another possibility is to use expectile regressions; an example of this approach applied in the context of Euro Area inflation can be found in Busetti et al. (2021).

the policymaker's task and preferences under risk management considerations, showing that the Fed's preferences during the Greenspan era placed greater emphasis on the risk of deflation than on the risk of excessive inflation. Similar characterizations of the Fed's behavior can be found in Gnabo and Moccero (2015), in terms of regime changes, and in Evans et al. (2015), in the context of uncertainty and the zero lower bound on nominal interest rates. The impact of the asymmetry of the distribution of inflation on the conduct of monetary policy is also examined using a quantile factor model in Fusari (2024) and a quantile MIDAS model in Ghysels (2018).

From a policymaker's perspective, this paper contributes to the ongoing discussion about how to communicate inflation forecast uncertainty. Inflation forecasting uncertainty is usually communicated through fan charts (see Franta et al., 2014, for a comprehensive overview of fan chart's use in central banks). Recently, the discussion was revived by Bernanke (2024), who based on weak grounds of the methodology, suggested that fan charts should be completely discarded from communications of central banks and replaced with a set of alternative scenarios, for example. This suggestion is opposed (Aikman and Barwell, 2024). For example, it has been argued that alternative scenarios are an even more ad hoc procedure than fan charts. This paper presents a possible response to concerns about the methodology behind fan charts.

The structure of the paper is as follows: The second section introduces the modelling approach and the data used for the estimations. Section 3 presents the results, starting with the quantile regression estimates and then moving on to the conditional distribution tail estimates. We then compare the estimated tails of the future inflation distribution to the ex-post observed inflation, and discuss the balance of inflation risks. Section 3 also includes an example of inflation risk forecasting. Section 4 concludes.

2. Models and Data

The analysis is based on quantile regressions. The quantiles of the future inflation distribution are conditional on the current macroeconomic variables (predictors), $\hat{Q}_\tau(\pi_{t+h}|x_t)$, and it is assumed the the relationship is linear:

$$\hat{Q}_\tau(\pi_{t+h}|x_t) = x_t\beta_\tau. \tag{1}$$

π_{t+h} denotes the annualized month-on-month headline CPI inflation h months ahead, and x_t is the vector of conditioning (risk) variables. The vector of parameters β_τ is quantile-specific, where τ refers to a particular quantile. We consider three quantiles: $\tau = 0.10$, 0.50, and 0.90, which capture the tails and center of the inflation outlook distribution. The tails represent risk. For example, quantile $\tau = 0.10$ represents the threshold below which inflation occurs, on average, once every 10 months (i.e. roughly once a year) given the observed macroeconomic variables.[4]

The estimation follows the standard approach introduced by Koenker and Bassett (1978). For each quantile, prediction errors are minimized using the check function. More precisely, the vector of parameters $\hat{\beta}_\tau$ is obtained:

[4] In the Inflation at Risk literature, the 10th and 90th percentiles are commonly used in the case of quarterly data. However, we do not adjust these percentiles for the monthly data due to the small number of observations in our sample (see the discussion on the number of observations in the tails below).

$$\hat{\beta}_\tau = \underset{\beta_\tau \in \mathbb{R}^k}{\arg\min} \sum_{t=1}^{T-h} \left[\rho_\tau\left(\hat{Q}_\tau(\pi_{t+h}|x_t) - x_t\beta_\tau\right)\right], \tag{2}$$

where $\rho_\tau(\cdot)$ is a check function defined as follows:

$$\rho_\tau(u) = u\left(\tau - \mathbf{1}_{\{u<0\}}\right), \tag{3}$$

with $\mathbf{1}_{\{u<0\}}$ denoting the indicator function.

We extend minimization (2) to include a non-crossing quantiles constraint, which ensures that the estimated tails and the center of the distribution do not intersect in the sample:

$$x_t\beta_{0.10} \leq x_t\beta_{0.50} \leq x_t\beta_{0.90}, \text{ for all } t = 1, \dots, T-h. \tag{4}$$

The solution to equations (2)-(4) follows Bondell et al. (2010). The non-crossing constraint is motivated by the short data sample in the Czech case. Adding the non-crossing restriction provides more information for estimation, as it takes the center of distribution into account when estimating tails. Therefore, this procedure addresses the issue of unstable estimates of conditional quantiles, thereby increasing the robustness of the estimate and presumably improving forecasts.[5]

To discuss the balance of inflation risks, we fit four estimated conditional quantiles $(\tau = 0.10, 0.25, 0.75, 0.90)$ using a skewed-*t* distribution (Azzalini and Capitanio, 2003):

$$f(\pi_{t+h}|\mu,\sigma,\alpha,\nu) = \frac{2}{\sigma} t\left(\frac{\pi_{t+h}-\mu}{\sigma};\nu\right) T\left(\alpha \frac{\pi_{t+h}-\mu}{\sigma}\sqrt{\frac{\nu+1}{\nu+\left(\frac{\pi_{t+h}-\mu}{\sigma}\right)^2}};\nu+1\right), \tag{5}$$

where the distribution parameters represent location (μ), scale (σ), shape (α) and fatness (ν) respectively. The fitting procedure minimizes the distance between the four conditional quantiles and the quantiles implied by the skewed *t*-distribution. This method was proposed by Adrian et al. (2019) and is widely used in the 'Growth at Risk' literature.

Using the fitted t-distribution, we can analyze the development of inflation risk based on the entire distribution, rather than just two quantiles. Conversely, fitting a skewed t-distribution imposes a specific unimodal distribution, thus discarding predictive distributions with multiple modes. As Mitchell et al. (2024) demonstrated for GDP growth, the multimodality of the predictive distribution of macroeconomic variables may need to be considered. In the case of inflation, for example, the two-regime model of inflation from Borio (2023) implies multimodality. This is why we only use the fitted t-distribution for computing the balance of inflation risks, but not for estimating the tails.

To discuss the in-sample and out-of-sample fit of quantile regression, several natural counterparts are considered. Firstly, unconditional sample quantiles are used to demonstrate that conditioning improves the fit. Secondly, OLS estimates of quantile specifications demonstrate how imposing symmetry on predictive distributions can lead to inaccurate results. Predictive distributions based on OLS are computed as an estimate of the conditional mean of the future inflation distribution,

[5] An illustration of the effect of non-crossing quantile constraint on estimated tails of conditional distribution can be found in Appendix A.5.

with the tails following a normal distribution and a standard deviation equal to the estimated standard deviation of the error term within the linear regression.

Finally, the GARCH model is also considered when forecasting is carried out. Compared to quantile regressions, which primarily draw on observations in the tails during estimation, the GARCH approach is less data-demanding. Furthermore, the GARCH model only requires the estimation of a few additional parameters. On the other hand, GARCH imposes symmetry on the predictive distribution. In its simplest form, GARCH(1,1), the specification is as follows:

$$\hat{Q}_\tau(\pi_{t+h}|x_t) = x_t\beta + \hat{Q}_\tau(\varepsilon_t|x_t), \tag{6}$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$ for $\sigma_t^2 = \omega^h + \gamma^h * \varepsilon_{t-1}^2 + \delta^h * \sigma_{t-1}^2$.

Using GARCH allows us to discuss whether second or third moments are important for forecasting the tails of the conditional distribution of inflation. The model is not discussed in in-sample exercises because, unlike quantile regressions, it does not provide a straightforward interpretation (for example, there is no guarantee that 10% of observations reside in the lower tail defined by the 10th percentile).

The models are estimated using data from the Czech Republic from January 2000 to December 2024. To assess the dynamics of inflation forecast risks on a regular and timely basis, and thus support the conduct of monetary policy, monthly data is required. All the variables used in the models presented below are available on a monthly basis, except for the nominal wage and output gap, which are interpolated to create monthly data. Spline interpolation is used.

The specification of quantile regressions in terms of the number of conditioning variables is subject to constraints in order to achieve accurate tail estimation. Chernozhukov and Fernández-Val (2011) suggest a rule of thumb for the quantile regression inference to reliably estimate the tails of the conditional distribution. The sample size relevant for the inference of the τ-th percentile (τT), adjusted for the number of covariates (k) in the model should exceed 15, i.e. $\frac{\tau T}{k} > 15$. Even with monthly data, our sample size implies $\tau T \sim 30$ suggesting two predictors. As the no-crossing condition adds some information from the center of the distribution to the tails, we work with three predictors and a constant.[6]

We take all the triplets from the set of potential predictors and chose the specification with the best in-sample fit. The fit is measured using a quantile score:

$$QS_\tau = \sum_{t=1}^{T-h} \left(\pi_{t+h} - \hat{Q}_\tau(\pi_{t+h}|x_t)\right)\left(\tau - \mathbf{1}_{\{\pi_{t+h} < \hat{Q}_\tau(\pi_{t+h}|x_t)\}}\right), \tag{7}$$

which is an objective function in (2). The quantile score is a quantile-specific representation of loss. Therefore, a lower value indicates a better fit for the quantile. Our benchmark specifications minimize the sum of quantile loss for both tails (i.e. for $\tau = 0.10, 0.90$).

[6] Another important consideration related to the number of predictors concerns the form of inflation. When inflation is defined as the year-on-year change in the CPI, one-off large shocks, which should not affect the relationship between the tail and the predictors, can significantly impact the results significantly because they affect 12 consecutive inflation figures. Consequently, two large shocks could constitute the sample relevant for the estimation of tails. This is why we work with month-on-month changes in the CPI, where a large shock implies a tail event lasting one period only.

To assess forecasting performance, we compute the average quantile score of direct forecasts for a given horizon h using rolling window estimations, starting with the window 2000M1–2015M6 (a forecast for 2015M6+h) and ending with the window 2000M1–2023M12 (a forecast for 2023M12+h). Note that we assess directly the forecasting performance of future inflation distribution tails. An alternative approach is to focus on prediction errors and model them using quantile regressions (Adams et al., 2021). This approach requires a model for the inflation mean/median forecast.

Conditioning variables are motivated by various theories of inflation, including possible nonlinearities and regime changes, as well as by the existing literature on Inflation at Risk. Most prominently we include variables that constitute real marginal costs in a small open economy Phillips curve i.e. the list of variables tested consists of measures of real economic activity – the output and unemployment gaps, industrial production gap, the measure of costs of labor and capital – the real wage gap, the real interest rate, and the import costs – the real exchange rate gap and eurozone inflation in the Czech koruna (CZK). We also include producer price index (PPI) growth as a proxy for supply shocks and oil prices, food prices and global producer prices in order to elaborate on the nature of supply shocks.

Furthermore, we consider the monetary aggregate (growth of real monetary aggregate M3) as suggested by the Quantity Theory of Money; real credit growth as a measure of financial conditions (López-Salido and Loria, 2024); the nominal monetary policy rate (the cost channel of monetary policy, see e.g. Chowdhury et al., 2006); the nominal exchange rate growth (Banerjee et al., 2024) and inflation momentum, which is defined as the difference in inflation from the previous period (behavioral macroeconomic models of inflation, see e.g. De Grauwe, 2011) which is in the context of Inflation at Risk considered in Szendrei and Bhattacharjee (2024). Finally, the numerical value of the inflation target is included in order to control for its changes over time and as a proxy for inflation expectations. A table showing all the variables used, including potential predictors, can be found in Appendix B.

3. Results

The estimation results are discussed in the following subsections. In line with the objective of describing inflation risks, we do not discuss the mean of the inflation conditional distribution in detail, but focus on the tails instead.

3.1 Quantile Regression Estimates

The in-sample fit of various model approaches and specifications shows that quantile regressions fit the tails of the distribution more accurately than unconditional quantiles and tails of predictive distributions based on OLS estimation (see Appendix A.2). Conditioning on current macroeconomic variables and accounting for the third and higher moments of the inflation outlook conditional distribution provides a better explanation of the observed inflation data.

Table 1: Estimated Coefficients, Full Sample

	Quantile regression (no-crossing)			OLS
Quantile:	*0.10*	*0.50*	*0.90*	
h = 1 month				
Constant	-1.54	1.71	5.53	1.78
	[-2.02,-1.19]	[1.44,2.05]	[5.13,6.24]	[1.37,2.19]
Real interest rate gap	-0.83	-0.65	-0.65	-0.69
	[-1.22,-0.58]	[-0.89,-0.41]	[-0.94,-0.42]	[-0.94,-0.45]
PPI growth	-0.05	0.28	0.60	0.38
	[-0.12,0.26]	[0.15,0.36]	[0.38,0.64]	[0.29,0.47]
Nominal interest rate gap	-0.35	-0.35	0.54	0.15
	[-1.17,0.17]	[-0.62,0.34]	[0.23,1.40]	[-0.38,0.68]
h = 6 months				
Constant	-1.22	1.82	6.42	2.04
	[-1.49,-0.90]	[1.54,2.13]	[5.06,6.78]	[1.62,2.47]
Real interest rate gap	-0.68	-0.55	-0.32	-0.69
	[-1.00,-0.55]	[-0.91,0.37]	[-0.99,-0.04]	[-0.95,-0.42]
Real wage gap	0.60	0.66	0.73	0.63
	[0.29,0.69]	[0.21,0.78]	[0.26,1.06]	[0.40,0.85]
PPI growth	-0.09	0.19	0.63	0.29
	[-0.21,0.09]	[0.08,0.26]	[0.35,0.70]	[0.20,0.38]
h = 12 months				
Constant	-1.68	2.16	6.68	2.24
	[-2.17,-1.09]	[1.65,2.41]	[5.53,7.03]	[1.80,2.69]
Real interest rate gap	0.44	0.58	0.82	0.63
	[-0.08,1.07]	[-0.05,0.95]	[-0.02,1.11]	[0.35,0.91]
Real wage gap	1.03	1.23	1.51	1.47
	[0.75,1.43]	[0.66,1.81]	[1.01,2.09]	[1.24,1.71]
PPI growth	0.04	0.28	0.42	0.35
	[-0.07,0.22]	[0.08,0.36]	[0.24,0.73]	[0.26,0.44]

Notes: No-crossing condition on 0.10^{th}, $0,50^{th}$ and 0.90^{th} quantiles imposed. 80% confidence bands are reported, 500 bootstrap samples used for inference.

Table 1 presents the estimation results of quantile regressions for three horizons ($h = 1,6,12$) with the no-crossing condition imposed. These regressions fit the tails best, as measured by quantile scores. The best specifications are close to the standard Phillips curve relationship with the real wage gap as an indicator of labor market slack and PPI growth as a proxy for supply shocks. In addition, the specifications include the real interest rate suggesting some role for monetary policy and inflation expectations.

The majority of the predictors are statistically significant with the sign suggested by the Philips curve.[7] Tighter labor market conditions are associated with an increase in inflation outlook distribution quantiles and the same is estimated for the supply shocks proxied by PPI growth. Additionally, the negative coefficient at the real interest rate gap (positive coefficients for the horizon of twelve months are not statistically significant) could reflect the link between monetary policy and future inflation distribution: tightening of monetary policy is associated with a fall in future inflation distribution percentiles.

[7] The confidence bands were estimated using Fitzenberger's (1997) moving block bootstrap method. The table presents centered 80% confidence bands to facilitate direct discussion of the statistical significance of the asymmetric effects of the predictors. If the confidence bands do not overlap, this implies that the 90% lowest values covered by one parameter's confidence band is lower than 90% of the highest values covered by the other parameter's confidence band. The statistical significance of the different parameter values can then be concluded.

More interestingly, asymmetry of the conditional distribution of future inflation related to some predictors is found. In this respect, the conditioning variables/predictors can be divided into two groups. First, there are variables that shift the entire conditional distribution. The parameters at these variables are statistically similar across quantiles. This is the case of the real interest rate gap and real wage gap. There are no nonlinearities in the relationship between the real interest rate, the real wage gap and future inflation.

Second, PPI growth and the nominal interest rate (for one-month horizon) are associated with an asymmetric conditional distribution of future inflation. PPI growth relates to minor changes in the lower tail, while the upper tail moves in the direction of price changes with an economically significant magnitude. This behavior could indicate regime-specific downward price rigidity: in a low-inflation environment, a slowdown in PPI growth does not lead to lower inflation. However, in periods of high inflation, changes in production factor prices are reflected in the final price. At a micro level, this finding may reflect changes in the frequency of price changes depending on the level of inflation (see the empirical evidence found in Karadi et al., 2024). The higher the inflation due to a PPI shock, the faster the pass-through of supply shocks.

Some evidence of the asymmetric effects of nominal interest rates is found in the specification with the real interest rate (for a one-month horizon). For the upper tail, the positive coefficient at the nominal interest rate can be interpreted as an absence of the immediate effect of monetary policy when both real and nominal rates change similarly and at the same time negative coefficient is estimated for the real interest rate.[8] For the center and lower tail of the inflation outlook distribution, the coefficient at the nominal interest rate is not statistically significant, implying the usual relationship between monetary policy and inflation. An increase in the (real) monetary policy rate is associated with a fall in inflation.

Table 1 also documents that common forecasting models, which provide a conditional mean or median forecast, could be misleading with regard to the balance of inflation risks. For example, the OLS estimate indicates a stronger association between current PPI growth and one-month-ahead inflation than the quantile regression suggests for the lower tail. If PPI growth slows down, the lower tail is not statistically significantly affected. Therefore, the inflation observed one month later will, on average, be higher than suggested by the conditional mean forecast – surprise in the observed inflation will be more likely on the upper side. In other words, although conditional mean/median models could suggest otherwise, PPI growth is not associated with downward inflation risks in the Czech economy.

In contrast to usual findings in the literature, we found no substantial role for financial conditions in the Czech Republic. Neither the real credit growth nor the real money balances improve the fit of the model at the tails of the inflation forecast distribution. Other studies have found that financial conditions vary with the lower tail (or both tails): López-Salido and Loria (2024) and Banerjee et al. (2024). When looking for the best-fitting three-variable specification, we included credit growth and money growth, but these variables were not chosen as they did not improve the in-sample fit (see Appendix A.2).

[8] A fall in the real interest rate due to an increase in inflation expectations (with no change in the nominal rate) is associated with a rise in the upper tail of one-month-ahead inflation.

Estimates of core inflation instead of the CPI inflation are presented in Appendix A.4. As with CPI inflation, the best in-sample fit specifications include real wage gap and PPI growth with expected signs. In addition, lagged inflation emerges as a predictor for all time horizons. Unlike CPI inflation, no predictor exhibits an asymmetric effect on the tails. This is particularly surprising in the case of PPI growth, given its strong relationship with the asymmetrical conditional distribution of headline CPI inflation.

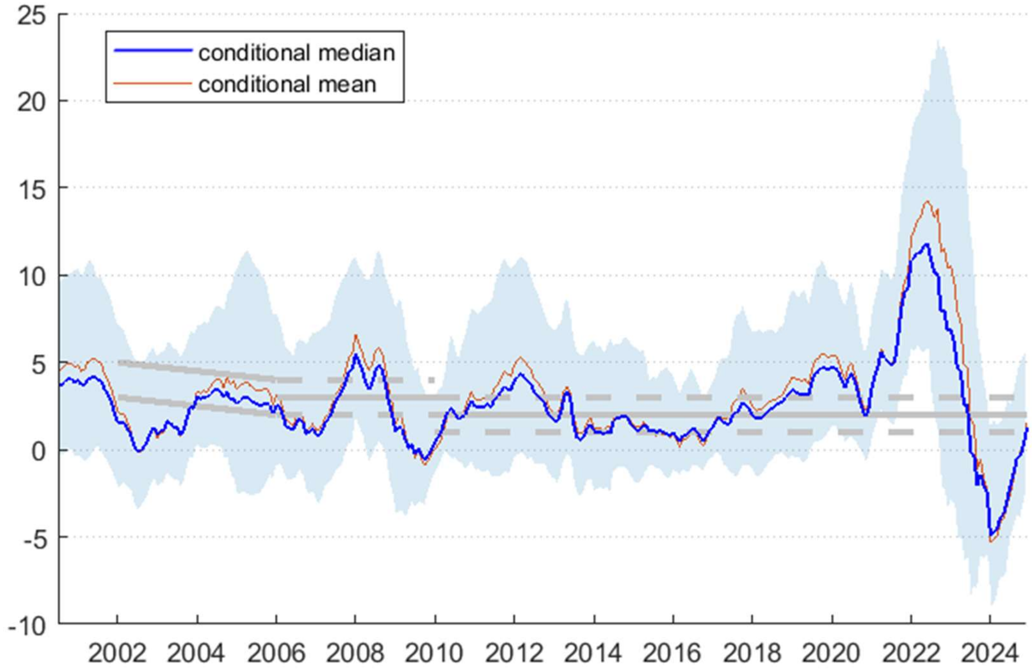
When the same specifications are compared for the core and the headline inflation rates (see Appendix A.4), a symmetric effect of the PPI on the core inflation rate is observed. This suggests that the asymmetry of headline CPI inflation outlook distribution is related to the energy and food price components of headline inflation, which are excluded from the core inflation.

Moreover, the variable selection procedure based on the in-sample fit selects PPI growth rather than its components: global food prices, global oil prices, Euro Area PPI, and the exchange rate. This suggests that PPI growth provides a better fit and serves as a summary measure of factors that are more relevant for the location and shape of the inflation outlook distribution.

3.2 Distribution of Inflation Outlook

The inflation outlook distribution provides policymakers with useful information. This is particularly important for a central bank that has an explicit numerical inflation target. The conditional distribution of the inflation forecast compared to the inflation target describes the risks of the inflation outlook. If the upper tail or both tails of the forecast distribution are above the target, this suggests the risk of overshooting the target, and vice versa. Figure 1 shows estimates of the upper and the lower tails, as well as the center (mean and median), of the month-on-month inflation outlook distribution. Furthermore, the inflation target and the tolerance band for year-on-year inflation are indicated.

Figure 1: Estimated Tails of Six-Months-Ahead Month-on-Month Annualized Inflation Together with the Conditional Median and Conditional Mean



Note: The blue area indicates the range between 0.10th and 0.90th quantiles. The thick grey lines indicate the inflation target and the tolerance band, both for year-on-year inflation. The model specification includes a constant, the real interest rate gap, the real wage gap and PPI growth. The year markers denote the start of each year.

For example, the forecasts of inflation tails for 2008 suggest a risk of overshooting the inflation target. In January 2008 (the forecast was made in 2007M6), for instance, the upper tail is well above the tolerance band, while the lower tail is close to the lower bound of the band. Consequently, the mean and median of the distribution are above the target. The opposite risk of target undershooting emerged in 2009. Both tails of the inflation distribution headed below the inflation target and its lower tolerance band. This risk arose as a consequence of the global financial crisis and falling foreign demand.

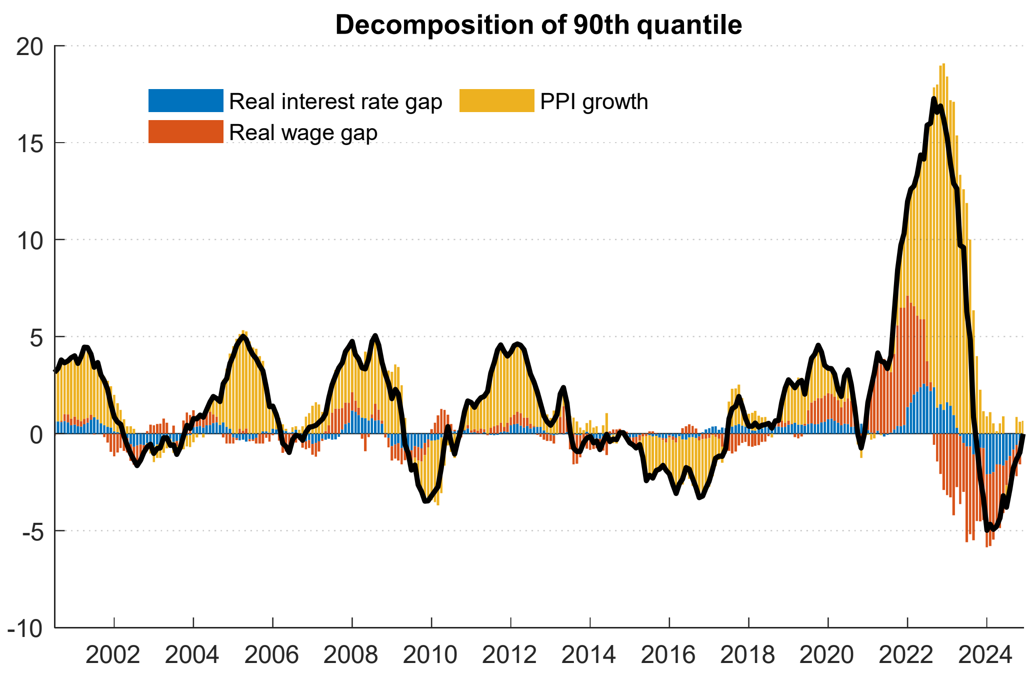
The upper tail of the inflation forecast distribution remained elevated in 2011 and 2012, before declining. It then gradually fell to the level of the target in 2016, indicating an increasing risk of inflation target undershooting. This decline coincided with a period during which the Czech National Bank expressed concerns about deflation. The zero lower bound on the nominal interest rate was reached in November 2012, after which the exchange rate commitment was launched in November 2013 to further ease monetary policy. The FX commitment was abandoned in April 2017, which coincides with a significant increase in the upper tail of the inflation outlook distribution, i.e. the forecasted upper tail being above the upper bound of the inflation tolerance band. Later in 2019 both tails of the inflation forecast distribution started to shift above the target suggesting rising the risk of inflation target overshooting. Recently, the entire distribution fell below the target again.

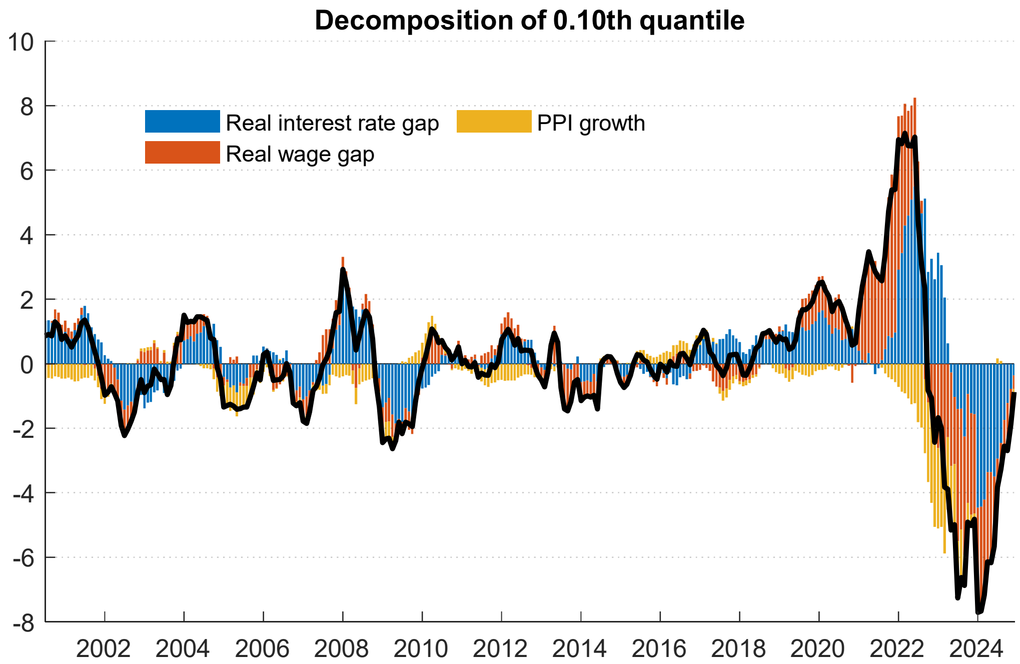
Apart from the center of the distribution and its tails, the distance between the tails also bears an important message. It relates to the second moment of the predictive distribution suggesting the extent of uncertainty associated with an inflation forecast. The greatest distance between the two tails is observed at the end of 2022, documenting extreme uncertainty related to inflation outlook following the energy price shocks of that year. On the other hand, the narrowest range is estimated for November 2016, when extremely low domestic inflation was accompanied by low imported inflation. A similar situation was observed at the beginning of 2010, when subdued domestic and foreign demand were accompanied by the European debt crisis.

Finally, the conditional distribution presented in Figure 1 is asymmetric, as indicated by the difference between the mean and the median. If a policymaker is provided with a conditional mean forecast, an asymmetric predictive distribution implies that ex-post observed inflation will be more often observed on one side of the point forecast than on the other.

A mean above the median suggests an upward skew towards higher inflation, whereas the opposite true when the distribution is skewed downwards. Upward skewness is found during the recent inflation surge starting in 2022, as well as in the period before the GFC. The opposite asymmetry is found in the period following the GFC's impact on the Czech economy and in the most recent period, which covers the first half of 2024. The reasons for this skewness are touched upon in the subsection 3.1 Quantile Regression Estimates and further analyzed using the decomposition of conditional distribution tails into the contributions of the respective predictors in Figure 2.

Figure 2: The Decomposition of the Upper $(\boldsymbol{\tau = 0.90})$ ***and Lower*** $(\boldsymbol{\tau = 0.10})$ ***Tails of the Six-Months-Ahead Month-on-Month Annualized Inflation Forecast Distribution into the Predictors***





Note: The constant is excluded from the decompositions. The model specification includes a constant, the real interest rate gap, the real wage gap and PPI growth.

It turns out that both tails have dropped swiftly recently driven mainly by a sharp decline in the real wage and real interest rate gaps. The real wage gap dropped significantly due to the sharp increase in the price level, which was not offset by an increase in the nominal wage. The decomposition suggests that monetary policy, i.e. the real interest rate gap, also pushed both tail forecasts down.

Due to the asymmetric effect on the predictive distribution, the role of current PPI growth is more significant for the upper tail than for the lower tail. When the PPI falls, as was observed in the second half of 2009 due to falling oil prices, for example, the whole distribution does not shift downwards, but becomes narrower. Consequently, the fall in oil prices did not pose a significant disinflationary risk to Czech inflation in 2009.

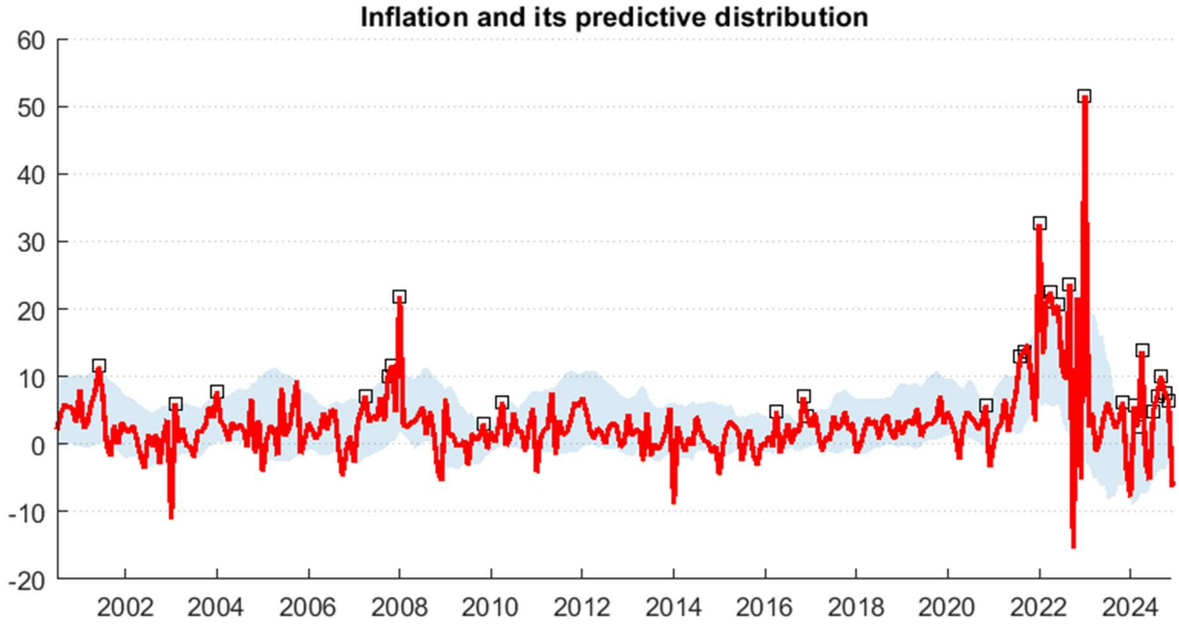
3.3 Observed Inflation vs. Predictive Distribution

The relative position of the ex-post observed inflation and the predictive distribution quantile provides several interesting insights. Figure 3 shows the observed inflation (red line) and the tails of the predictive distribution of inflation (the range between 10^{th} percentile and 90^{th} percentile indicated by the blue area). In general, we are interested in periods of 'tail events' lasting more than one period (month) because a one-month episode could simply be the results of a large shock. Consecutive periods of inflation lasting several months and observed at the tail of the predictive distribution, which are caused by independent shocks, are of very low probability. Thus, their appearance may suggest a change in inflation dynamics or a non-normal distribution of shocks.[9] A

[9] For example, the probability of two consecutive independent shocks in row from the first decile of a normal distribution is $0.1 * 0.1$, i.e. once every 100 periods. The probability of three consecutive 'tail' shocks can be expressed as an event that happens once every 1,000 periods. In our sample, we observe five episodes of at least two consecutive months of an upper tail event representing a much higher occurrence than that suggested by normally distributed independent shocks.

regime change may capture changes in expectation formation, non-linearity in the relationship between spare real economy capacity and inflation, etc.[10]

Figure 3: The Estimated Tails of Six-Months-Ahead Inflation Together with the Observed Month-on-Month Annualized Inflation



Note: The model specification includes a constant, the real interest rate gap, the real wage gap and PPI growth. Actual inflation realizations above 90th percentile are denoted by squares.

The majority of the tail events shown in Figure 3 are one-off episodes. An example of a one-off event is January 2008, when actual inflation was above the upper tail. This was a period of tax changes, during which time headline inflation was well above the target, but the CNB did not respond to this given the one-off nature of the inflation increase.

Several months of observed inflation above the upper tail of its predictive distribution are observed in the second half of 2021 when the CNB started to normalize its monetary policy by increasing the monetary policy rate by 25 basis points. Finally, also the recent period starting by 2024M1 witness the upper tail event lasting several months although the distribution is heading below the target.

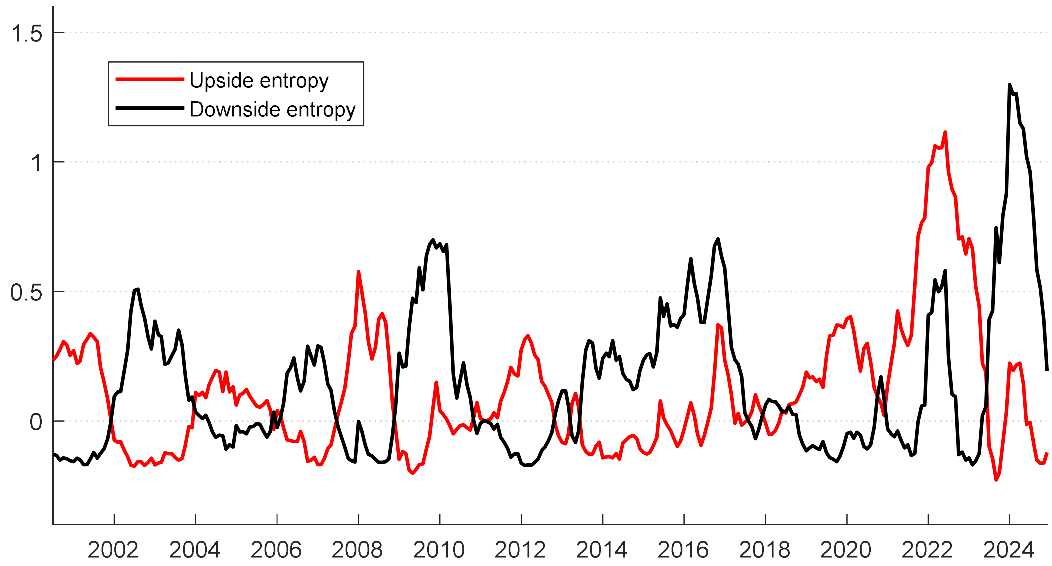
3.4 Balance of Inflation Risks

In addition to the absolute position of the predictive distribution and its width, its asymmetry is of interest to policymakers, particularly those acting as risk managers. An asymmetric predictive distribution provides guidance on whether large positive surprises in ex-post observed inflation are more likely than large negative surprises, or vice versa. Density forecasts provide more complex information than point forecasts.

[10] López-Salido and Loria (2024) show that the regime-switching model of US inflation produces inflation regimes that correspond to the observed inflation residing in the tails of its predictive distribution.

Following Adrian et al. (2019), we compute the upside and downside entropies to measure upside and downside inflation risks. Upside (downside) entropy measures the probability mass above (below) the conditional median of the distribution of future inflation relative to the mass of the unconditional distribution. An increase in upside entropy, for example, indicates higher probability of inflation values exceeding their historically observed frequency.

Figure 4: Inflation Risks. Quantile Regression Estimation with No-Crossing Constraints



Note: The model specification with six-months-ahead inflation and a constant, real interest rate gap, real wage gap and PPI growth is used to estimate upside entropy and downside entropy.

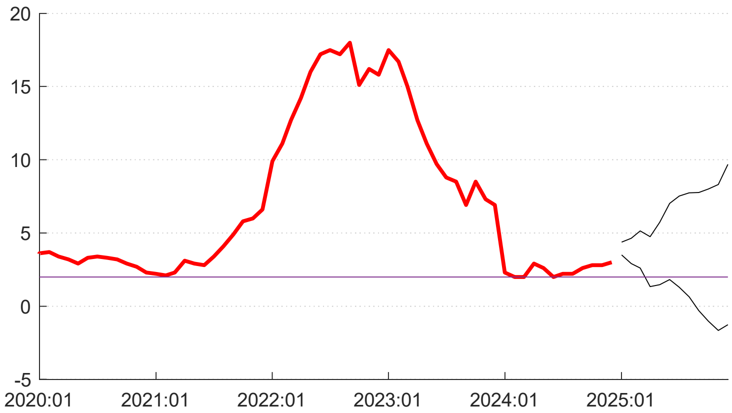
Note that the computation of upside and downside entropy requires the use of entire distributions. We therefore employ skewed *t*-distributions that are fitted to a set of conditional quantiles. Figure 4 presents the two measures of inflation risks. Three episodes of heightened downside risk to inflation are estimated. The first relates to 2009–2010, when the GFC and the European debt crisis had a heavy impact on the Czech economy. At the end of 2008, the CNB clearly stated the balance of risks on the downside when forecasting for 2009 (CNB, 2008). The second episode is even more extreme and covers the period of the zero lower bound, exchange rate commitment and deflation concerns. Finally, the period at the beginning of 2024 (i.e. the forecast is made in summer 2023) is also experiencing heightened downside inflation risks. Upside inflation risks can be observed in the wake of the GFC at the beginning of 2008, as well as during the first half of 2022 amid energy price shocks. In addition to the increased risk of high inflation, as indicated by the rising upside entropy in 2022, there has also been an increase in downside entropy, albeit to a lesser extent. This indicates rising uncertainty about the inflation outlook, which is consistent with the widening of the predictive distribution as discussed in the previous subsection.

3.5 Forecasting

The best specifications for forecasting differ depending on the forecasting horizon. However, in general they are similar in terms of the predictors they include (see Appendix A.3), with PPI growth, lagged inflation, the real wage gap and the unemployment gap being among the most frequent. For each horizon, the out-of-sample fit of the best specification is compared to two 'naïve' benchmarks: OLS estimation with a predictive distribution based on the estimated standard deviation of errors and unconditional quantiles. Moreover, the GARCH(1,1) model is considered to examine the role of various conditional distribution moments in forecasting.

In Appendix A.3, we show that for lower tail, quantile regressions provide forecasting performance that is comparable to that of OLS estimation. For the upper tail, GARCH(1,1) provides the best fit. Therefore, the explicit modelling of the third moment of the inflation outlook distribution (skewness), or of higher moments, is not crucial for forecasting. The second moment (variance), whether time-invariant as in OLS estimation or time-varying as in the GARCH model, is sufficient for forecasting the tails of inflation outlook distribution.

Figure 5: Forecasts of 10^th^ and 90^th^ Percentile of Year-on-Year Changes in the CPI



Note: The inflation target is indicated by the horizontal line. 10^{th} percentile is forecasted using quantile regression with a no-crossing constraint and 90^{th} percentile is forecasted using GARCH(1,1) model.

Figure 5 shows the forecasted 10^{th} and 90^{th} percentiles of the CPI inflation outlook distribution. The lower tail is based on quantile regressions, while the upper tail is based on GARCH models. The forecast of tails for year-on-year inflation distribution is compiled as follows. For each horizon, a direct forecast of inflation distribution tails h periods ahead is computed, using the best fitting specifications for month-on-month inflation. Given the last observed CPI value, CPI outlooks CPI corresponding to the forecasted percentiles are computed. Using these indexes, we obtain forecasts of the percentile of year-on-year changes in CPI. We focus on year-on-year changes as these are of primary interest to policymakers.

It turns out that, in the short term, there is a substantial risk of the inflation target being overshot, as the 10th percentile exceeds the target. Looking at the entire one-year horizon, upside inflation risks can be concluded.

4. Conclusions

Our paper demonstrates the applicability of the Inflation at Risk (IaR) approach to the forecasting and assessment of inflation risks using the Czech economy as a case study. Using quantile regressions with non-crossing constraints, we modelled the tails of the inflation distribution and highlighted the inherent asymmetries in inflation risks. Compared to traditional symmetric fan charts, this approach better captures the distributional nuances of inflation risks, offering central banks a more robust framework.

Our results underscore that the asymmetry of inflation risk distribution is related to producer price index (PPI) growth. These findings confirm the presence of potential nonlinearities in the traditional Phillips curve relationship and suggest that shifts in supply-side pressures have a disproportionate effect on inflation's upper tail in high-inflation regimes. Quantile regression models are selected based on their in-sample and out-of-sample forecasting performance. Furthermore, the application of IaR model to historical data allows for an insightful examination of the inflation risk distribution in response to external shocks and policy changes. Key event, such as the 2008 financial crisis, the introduction of the exchange rate commitment in 2013, and the energy price shocks of 2022, reveal inflationary pressures in line with prevailing narratives.

Lastly, this study highlights the practical benefits of the IaR framework. Quantile-based risk metrics, such as upside and downside entropies, provide a clearer picture of the risks associated with inflation forecasts, enabling policymakers to engage in more effective discussions about potential inflationary or deflationary pressures. For central banks, the IaR framework provides a viable means of quantifying and conveying inflation risks with greater clarity and robustness. The IaR approach provides central banks with a nuanced and adaptable tool for managing inflation risks.

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Appendix A: Additional Results and Robustness Checks

A.1 Covid Observations

In the benchmark estimation, COVID observations are included. However, the literature suggests that such observations can affect results substantially – see the discussion in Lenza and Primiceri (2022). Therefore, as a robustness check we drop COVID observations. More precisely, rows of data matrix $[\pi_{t+12}, x_{t-1}]$, for which the inflation π_{t+12} covers period $2020M4{:}\,2020M12$, are ignored during estimation. In forecasting assessment, the forecasts for months covering the period COVID are ignored.

Tables A1 and A2 show that estimation results are not affected much by the exclusion of COVID observations. However, as these observations can be viewed as a tail event, we prefer to include them in the estimation.

Table A1: Quantile Scores and Specifications, Sample without COVID Observations

	QRs with no-crossing condition	Unconstrained QRs	OLS	Unconditional quantiles
h = 12 months				
10th	183,48	183,48	219,32	201,84
	Constant, Lagged inflation, Output gap, Nominal interest rate gap	Constant, Lagged inflation, Output gap, Nominal interest rate gap	Constant, Production gap, Unemployment gap, Nominal interest rate gap	
90th	257,08	248,29	282,81	326,20
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	
Balance	446,79	438,10	507,85	570,00
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	
h = 6 months				
10th	182,69	181,18	217,49	211,61
	Constant, Real interest rate gap, Real wage gap, Euro Area inflation in CZK	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, Nominal exchnage rate growth	
90th	241,43	241,13	267,79	345,86
	Constant, Lagged inflation, Real wage gap, PPI growth	Constant, Real wage gap, PPI growth, Nominal interest rate gap	Constant, Lagged inflation, Real wage gap, PPI growth	
Balance	433,82	429,51	495445,00	561,00
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Lagged inflation, Real interest rate gap, Real wage gap	
h = 1 month				
10th	185,06	183,88	222,84	216,63
	Constant, Real interest rate gap, Real wage gap, Nominal interest rate gap	Constant, Real interest rate gap, Real wage gap, Euro Area PPI growth	Constant, Real interest rate gap, Oil price (in EUR) growth, Inflation momentum	
90th	226,25	225,16	252,47	349,96
	Constant, Real interest rate gap, PPI growth, Inflation momentum	Constant, Real interest rate gap, PPI growth, Inflation momentum	Constant, Real interest rate gap, PPI growth, Inflation momentum	
Balance	422,53	416,29	485,72	569,00
	Constant, Real interest rate gap, PPI growth, Nominal interest rate gap	Constant, Real interest rate gap, Real exchange rate gap, PPI growth	Constant, Real interest rate gap, PPI growth, Inflation momentum	

Note: The rows "Balance" show the quantile score for both 10th and 90th percentiles.

Table A2: Estimated Coefficients, Sample without COVID Observations

Quantile:	*0.10*	*0.50*	*0.90*	OLS
h = 1 month				
Constant	-1.50	1.70	5.47	1.77
	[-2.00,-1.15]	[1.37,1.96]	[4.89,6.11]	[1.36,2.19]
Real interest rate gap	-0.85	-0.67	-0.67	-0.70
	[-1.15,-0.56]	[-0.90,-0.39]	[-0.98,-0.40]	[-0.95,-0.45]
PPI growth	-0.05	0.27	0.60	0.38
	[-0.12,0.26]	[0.16,0.35]	[0.40,0.65]	[0.29,0.47]
Nominal interest rate gap	-0.27	-0.27	0.59	0.22
	[-1.13,0.13]	[-0.60,0.39]	[0.16,1.49]	[-0.34,0.78]
h = 6 months				
Constant	-1.21	1.82	6.44	2.10
	[-1.54,-0.91]	[1.54,2.19]	[5.09,7.04]	[1.67,2.54]
Real interest rate gap	-0.70	-0.56	-0.33	-0.7024
	[-1.04,-0.57]	[-0.89,-0.33]	[-1.00,0.01]	[-0.97,-0.43]
Real wage gap	0.60	0.66	0.72	0.63
	[0.29,0.73]	[0.23,0.81]	[0.26,1.02]	[0.40,0.86]
PPI growth	-0.09	0.19	0.62	0.28
	[-0.22,0.08]	[0.09,0.26]	[0.35,0.69]	[0.19,0.37]
h = 12 months				
Constant	-1.56	2.15	6.73	2.31
	[-2.16,-1.08]	[1.69,2.50]	[5.73,7.33]	[1.85,2.77]
Real interest rate gap	0.39	0.57	0.82	0.61
	[-0.11,0.92]	[0.02,0.93]	[0.10,1.19]	[0.33,0.90]
Real wage gap	0.12	1.27	1.53	1.49
	[0.80,1.55]	[0.67,1.81]	[0.90,2.03]	[1.25,1.73]
PPI growth	0.05	0.28	0.42	0.35
	[-0.05,0.23]	[0.08,0.36]	[0.27,0.84]	[0.25,0.44]

Notes: No-crossing condition on 0.10^{th}, 0.50^{th} and 0.90^{th} quantiles imposed. 80% confidence bands are reported, 500 bootstrap samples used for inference.

A.2 In-Sample Fit

This appendix presents additional estimation results related to the in-sample fit of the quantile regression models. Table A3 shows the quantile scores for the best-fitting specifications of quantile regressions, OLS and unconditional quantiles. In the main text, the employed specifications are those, which fit both tails (i.e. both 0.10^{th} and 0.90^{th} quantiles) – the quantile score is presented in the rows '*Balance*'.

The fit of tails is best for quantile regressions. The quantile regressions with no-crossing constraint exhibit slightly worse fit, which is not surprising as every additional constraint to the optimization procedure in (2) decreases the fit. However, we stick to the constrained quantile regressions in the analysis due to lower volatility of estimated conditional quantiles (see Appendix A.5 for details).

Next, unconditional empirical quantiles explain lower tail more accurately than the OLS regressions and vice versa, the upper tail is explained more by the OLS in comparison to unconditional sample quantiles. This points to the different nature of lower and upper tails.

Table A3: Quantile Scores and Specifications, Full Sample

	QRs with no-crossing condition	Unconstrained QRs	OLS	Unconditional quantiles
h = 12 months				
10th	194.56	194.56	232.86	216.71
	Constant, Lagged inflation, Output gap, Nominal interest rate gap	Constant, Lagged inflation, Output gap, Nominal interest rate gap	Constant, Inflation target, Production gap, Nominal interest rate gap	
90th	268.74	261.30	293.50	351.46
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	
Balance	473.65	456.74	532.14	568.17
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	
h = 6 months				
10th	192.26	191.30	226.64	220.37
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Unemployment gap, Real interest rate gap, Real wage gap	
90th	252.32	251.86	279.51	353.29
	Constant, Real wage gap, Euro Area inflation in CZK, PPI growth	Constant, Real wage gap, Euro Area inflation in CZK, PPI growth	Constant, Lagged inflation, Real weage gap, PPI growth	
Balance	452.14	450.08	514.50	573.66
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Lagged inflation, Real exchange rate gap, Real weage gap	
h = 1 month				
10th	192.88	192.13	227.57	222.80
	Constant, Real interest rate gap, Real wage gap, Inflation momentum	Constant, Real interest rate gap, Real wage gap, Euro Area PPI growth	Constant, Output gap, Real interest rate gap, Inflation momentum	
90th	230.78	229.50	257.83	355.41
	Constant, Real exchange rate gap, PPI growth, Inflation momentum	Constant, Real exchange rate gap, PPI growth, Interest rate gap	Constant, Real interest rate gap, PPI growth, Inflation momentum	
Balance	433.05	427.80	495.98	578.21
	Constant, Real interest rate gap, PPI growth, Nomina interest rate gap	Constant, Real interest rate gap, Real exchange rate gap, PPI growth	Constant, Real interest rate gap, PPI growth, Inflation momentum	

Note: The rows "Balance" show the quantile score for both 10th and 90th percentiles.

To assess the role of credit and monetary aggregates, a robustness check with respect to the period starting by 2003M1 is carried out. Both monetary aggregate and credit are available starting by the year 2003. It turns out that financial/monetary variables do not enter the best fitting specifications (Table A4).

Table A4: Quantile Scores and Specifications, 2003+ Sample

	QRs with no-crossing condition	Unconstrained QRs with smoothing	OLS	Unconditional quantiles
h = 12 months				
10th	171.58	171.58	208.37	190.62
	Constant, Lagged inflation, Output gap, Nominal interest rate gap	Constant, Lagged inflation, Output gap, Nominal interest rate gap	Constant, Real interest rate gap, Real wage gap, Euro Area PPI growth	
90th	243.14	236.26	269.44	326.36
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	
Balance	422.64	415.58	480.79	516.99
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	
h = 6 months				
10th	168.35	167.77	203.97	193.36
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, Nominal exchange rate growth	
90th	231.07	231.13	257.86	329.52
	Constant, Lagged inflation, Real wage gap, PPI growth	Constant, Lagged inflation, Real wage gap, PPI growth	Constant, Lagged inflation, Real wage gap, PPI growth	
Balance	406.45	405.48	469.68	522.88
	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Real wage gap, PPI growth	Constant, Lagged inflation, Real interest rate gap, Real wage gap	
h = 1 month				
10th	170.23	168.63	203.24	194.91
	Constant, Real interest rate gap, Real wage gap, Inflation momentum	Constant, Lagged inflation, Real interest rate gap, Oil prices (in EUR) growth	Constant, Real interest rate gap, Oil prices (in EUR) growth, Inflation momentum	
90th	211.43	210.46	235.98	332.38
	Constant, Real exchange rate gap, PPI growth, Inflation momentum	Constant, Real exchange rate gap, PPI growth, Inflation momentum	Constant, Real interest rate gap, PPI growth, Inflation momentum	
Balance	388.86	384.03	449.89	527.29
	Constant, Real interest rate gap, PPI growth, Inflation momentum	Constant, Real exchange rate gap, PPI growth, Inflation momentum	Constant, Real interest rate gap, PPI growth, Inflation momentum	

Note: The rows "Balance" show the quantile score for both 10th and 90th percentiles.

A.3 Out-of-Sample Fit

Table A5 lists the best specifications for forecasting at a particular horizon and quantile scores from the out-of-sample forecasting exercise described in Section 3.5.

Table A5: Specifications for Forecasting and Their Quantile Scores in Rolling Window Exercise

	QRs with no-crossing condition	Unconstrained QRs	OLS	GARCH(1,1)	Unconditional quantiles
h = 12 months					
10th	1.13	1.06	1.10	1.33	1.23
	Constant, Unemployment gap, Real exchange rate gap, PPI growth	Constant, Production gap, Euro Area inflation in CZK, Nominal interest rate gap	Constant, Output gap, Unemployment gap, Interest rate gap	Constant, Real wage gap, PPI growth, Oil prices (in EUR) growth	
90th	1.55	1.75	1.72	1.45	2.75
	Constant, Unemployment gap, Euro Area inflation in CZK, Food prices growth	Constant, Production gap, Real wage gap, Nominal interest rate gap	Constant, Real interest rate gap, Oil prices (in EUR) growth, Euro Area PPI growth	Constant, Lagged inflation, Output gap, Real interest rate gap	
Balance	2.92	2.93	3.11	2.80	3.98
	Constant, Unemployment gap, Euro Area inflation in CZK, Food prices growth	Constant, Production gap, Real wage gap, Nominal interest rate gap	Constant, Unemployment gap, Real wage gap, PPI growth	Constant, Lagged inflation, Real wage gap, Real interest rate gap	
h = 6 months					
10th	1.00	1.01	0.98	1.10	1.16
	Constant, Unemployment gap, Real wage gap, Euro Area inflation in CZK	Constant,Real wage gap, Oil prices (in EUR) growth, Food prices growth	Constant, Output gap, Real wage gap, Euro Area inflation	Constant, Nominal interest rate gap, Real wage gap, PPI growth	
90th	1.36	1.32	1.21	1.09	2.61
	Constant, Real exchange rate gap, Euro Area inflation in CZK, Food prices growth	Constant, Real interest rate gap, Real exchange rate gap, Nominal interest rate gap	Constant, Real interest rate gap, Nominal interest rate gap, Euro Area PPI growth	Constant, Nominal interest rate gap, Real interest rate gap, Inflation momentum	
Balance	2.55	2.60	2.69	2.31	3.77
	Constant, Unemployment gap, Real wage gap, PPI growth	Constant, Real interest rate gap, Nominal interest rate gap, Oil prices (in EUR) growth	Constant, Real interest rate gap, Oil prices (in EUR) growth, Food prices growth	Constant, Nominal interest rate gap, Real interest rate gap, Real wage gap	
h = 1 month					
10th	1.01	1.04	0.98	1.10	1.13
	Constant, Unemployment gap, Real wage gap, Euro Area inflation in CZK	Constant, Output gap, Real wage gap, Oil prices (in EUR) growth	Constant, Output gap, Unemployment gap, Real wage gap	Constant, Output gap, PPI growth, Inflation momentum	
90th	1.18	1.19	1.20	1.11	2.48
	Constant, Production gap, PPI growth, Nominal exchange rate growth	Constant, Real interest rate gap, PPI growth, Nimonal nterest rate gap	Constant, Real interest rate gap, PPI growth, Inflation momentum	Constant, Real interest rate gap, Euro Area PPI growth, Food prices growth	
Balance	2.35	2.40	2.43	2.31	3.60
	Constant, Unemployment gap, Real exchange rate gap, PPI growth	Constant, Real interest rate gap, PPI growth, Nominal interest rate gap	Constant, Real interest rate gap, PPI growth, Output gap	Constant, Output gap, Real interest rate gap, PPI growth	

Note: The rows "Balance" show the quantile score for both 10th and 90th percentiles.

A.4 Core Inflation

This section presents resuts for the core inflation. Table A6 shows quantile regression estimates for best specifications according to the in-sample fit. In addition, Table A7 reports estimation results for core inflation for specification, which fits the best model of headline inflation. The purpose is to compare estimates for headline and core inflation for the same specification. The comparison confirms the fact that PPI growth relates to asymetry of headline inflation outlook while for the core inflation outlook the PPI growth lacks the asymetric influnce. It follows that the asymetric relation of PPI to headline inflation outlook distribution is linked to energy and food prices.

Table A6: Estimated Coefficients, Core Inflation, Full Sample

Quantile:	*0.10*	*0.50*	*0.90*	OLS
h = 1 month				
Constant	-1.87	0.33	3.12	0.33
	[-2.44,-1.7]	[-0.04,0.84]	[2.49,3.83]	[0.08,0.58]
Lagged inflation	0.26	0.27	0.50	0.36
	[0.08,0.38]	[0.17,0.49]	[0.26,0.61]	[0.29,0.44]
Real wage gap	0.29	0.28	0.47	0.40
	[-0.17,0.57]	[0.01,0.57]	[0.13,0.60]	[0.30,0.49]
PPI growth	0.12	0.24	0.21	0.23
	[0.02,0.27]	[0.03,0.30]	[0.07,0.32]	[0.17,0.28]
h = 6 months				
Constant	-1.59	0.77	3.69	0.72
	[-2.24,-1.27]	[0.34,1.10]	[2.62,4.69]	[0.45,0.95]
Lagged inflation	0.16	0.23	0.48	0.30
	[0.06,0.30]	[0.11,0.37]	[0.11,0.73]	[0.22,0.39]
Real wage gap	0.34	0.65	0.98	0.85
	[0.06,0.80]	[0.23,1.13]	[0.25,1.27]	[0.75,0.96]
PPI growth	0.13	0.13	0.14	0.15
	[0.02,0.22]	[-0.01,0.22]	[-0.02,0.26]	[0.09,0.21]
h = 12 months				
Constant	-1.60	0.97	5.29	1.14
	[-1.96,-1.11]	[0.55,1.41]	[3.73,5.71]	[0.86,1.42]
Lagged inflation	0.21	0.22	0.22	0.29
	[0.06,0.30]	[0.11,0.28]	[0.17,0.40]	[0.24,0.35]
Unemployment gap	0.29	1.14	3.91	1.54
	[-0.36,1.28]	[-0.22,1.93]	[0.94,4.67]	[1.04,2.04]
Real wage gap	0.60	0.80	0.87	0.96
	[0.24,1.13]	[0.29,1.23]	[0.56,1.46]	[0.86,1.07]

Notes: No-crossing condition on 0.10^{th}, 0.50^{th} and 0.90^{th} quantiles imposed. 80% confidence bands are reported, 500 bootstrap samples used for inference.

Table A7: Estimated Coefficients for Quantile Regression with Non-Crossing Condition, Full Sample, h=6 Months

	Headline inflation			Core inflation		
Quantile:	*0.10*	*0.50*	*0.90*	*0.10*	*0.50*	*0.90*
Constant	-1.22	1.82	6.42	-1.27	0.99	4.68
	[-1.49,-0.90]	[1.54,2.13]	[5.06,6.78]	[-1.92,-1.02]	[0.73,1.37]	[3.50,5.44]
Real interest rate gap	-0.68	-0.55	-0.32	-0.35	-0.35	-0.29
	[-1.00,-0.55]	[-0.91,0.37]	[-0.99,-0.04]	[-0.71,-0.07]	[-0.74,-0.12]	[-0.79,0.17]
Real wage gap	0.60	0.66	0.73	0.08	0.39	0.80
	[0.29,0.69]	[0.21,0.78]	[0.26,1.06]	[-0.32,0.74]	[-0.14,0.91]	[0.14,1.14]
PPI growth	-0.09	0.19	0.63	0.16	0.19	0.28
	[-0.21,0.09]	[0.08,0.26]	[0.35,0.70]	[0.05,0.25]	[0.03,0.29]	[0.12,0.41]

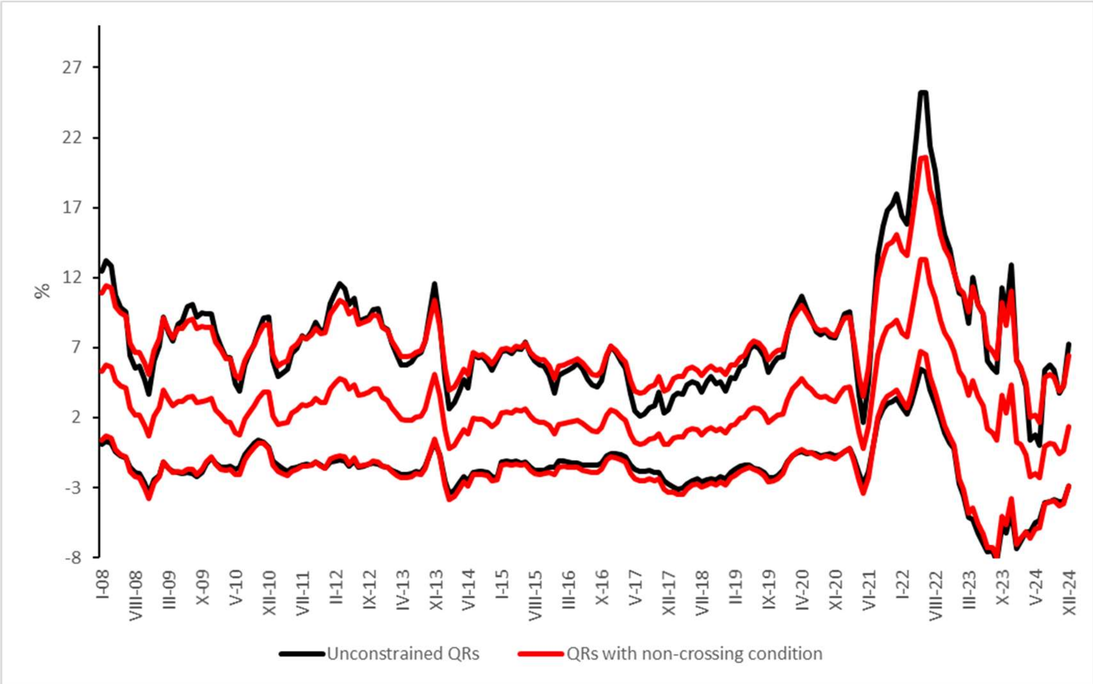
Notes: No-crossing condition on 0.10th, 0.50th and 0.90th quantiles imposed. 80% confidence bands are reported, 500 bootstrap samples used for inference.

A.5 Non-Crossing Quantile Constraint

This section discusses the effect of imposing non-crossing quantile constraint. In comparison to unconstrained quantile regression, the non-crossing quantile constraint lowers the in-sample fit because the constraint represents just an additional constrain in the minimization task (2) and the fit coincides with the value of minimum. For the out-of-sample fit, such consideration no longer holds and some specifications with imposed constraint exhibit better fit than unconstrained quantile regressions (Table A5 in Appendix A.5).

Furthermore, the volatility of the fitted quantiles is often lower if the constraint is employed. As an example, Figure A1 shows fitted tails based on unconstrained quantile regressions (black curves) and tails an center of distribution based on quantile regressions with non-crossing quantile constraint (red curves). The upper tail based on the estimate of unconstrained quantile regression is more volatile. Next, the difference between tails based on the two approaches can be economically significant. For example, the difference in upper tails exceeds 1.5 percentage points in the second half of 2017.

***Figure A1:** Fitted 0.10th and 0.90th Quantiles of 12-Months-Ahead Inflation for Different Estimation Approaches, in Addition, 0.50th Quantile for Quantile Regression with Non-Crossing Quantile Constraint is Presented*



Note: The model specification with constant, real interest rate gap, real wage gap and PPI growth is employed.

Appendix B: Data

The table below presents variables used in the paper. It also includes potential predictors tested to forecast the inflation distribution. All variables are related to the Czech Republic unless otherwise noted. HP filter used to de-trend some of variables below is two-sided in in-sample simulations. One-sided version of the HP filter is used in out-of-sample simulations.

Variable	**Description**	**Source**
Inflation measures		
Headline inflation	Headline CPI inflation, in percent m-o-m s. a. and annualized	CZSO
Core inflation	Headline CPI inflation excluding energy, fuel, and food, in percent m-o-m s. a. and annualized	CNB
Real economic activity		
Industrial production	Gap of industrial production index, percent deviation from the HP filter trend (lambda 14400)	CZSO and authors computation
Output gap	Gap of real GDP, percent deviation from a trend	CNB's Monetary Policy Reports
Unemployment gaps	Unemployment rate, percent deviation from the HP trend	CZSO Labor Market Survey and
Eurozone output gap	Gap of real GDP of eurozone (19 countries), percent deviation from a trend	
Supply costs measures		
Nominal interest rate gap	3M PRIBOR, percent deviation from the HP filter trend (lambda 14400)	CNB and authors computation
Real interest rate gap	3M PRIBOR adjusted by actual headline y-o-y inflation 6 months ahead, percent deviation from the HP trend (lambda 14400)	CNB and authors computation
Eurozone inflation	Headline HICP inflation, percent y-o-y	Eurostat
Eurozone PPI inflation	Effective eurozone PPI, percent y-o-y	Eurostat and CNB's Monetary Policy Reports
PPI inflation	Producer price index, percent m-o-m s. a.	CZSO
Nominal exchange rate depreciation	CZK/EUR depreciation, percent m-o-m annualized	CNB and authors computation
Real wage gap	Nominal average wage deflated by headline CPI, percent deviation from the HP filter trend (lambda 14400)	CZSO and authors computation
Real exchange rate gap	The real exchange rate computed based on CZK/EUR exchange rate and Czech CPI and headline HICP eurozone (19 countries)	CNB, CZSO, Eurostat and authors computation
Nominal credit growth	Credit to private sector, percent y-o-y	CNB
Real credit growth	Credit to private sector adjusted by headline CPI inflation, percent y-o-y	CNB and authors computation
Brent Oil Price	Global price of Brent crude oil, EUR per barrel	FRED (POILBREUSDM) converted to EUR using USD/EUR rate
World food price index	World food price index, percent m-o-m, s.a. and annualized	FRED (PFOODINDEXM)
Other measures		
M3 growth	M3 growth, percent y-o-y	CNB
Real M3 growth	M3 growth adjusted by headline CPI inflation, percent y-o-y	CNB and authors computation
Inflation momentum	Change of y-o-y inflation, p. p.	CNB and authors computation

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